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Non-singlet structure function at low and high x in the next-to-leading order and CCFR neutrino data

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Abstract . We find the complete solutions of the DGLAP equations for non-singlet structure function at low and high x using Lagrange's auxiliary method in the next-to-leading order and find analytical expressions for xF_3. Through our formalism, we have been able to separate out qualitatively low and high x regimes of deep inelastic neutrino scattering. Comparison with CCFR neutrino data as well as exact solutions is also made

Keywords : Non-singlet structure functions, low and high x, next-to-leading order.

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1. Introduction

Recently [1] we reported the complete solutions of DGLAP equations [2] in the leading order for non-singlet structure functions at low and high x. Using the Lagrange's auxiliary methods [3], we made phenomenological analysis with the CCFR data [4] with reasonable success. The formalism enables one qualitatively to separate the low and high x regions explored in deep inelastic neutrino scattering.

In the present paper, we incorporate the next-to-leading order [NLO] effects in the formalism and re-analyze the CCFR data.

Despite the maturity of the field and its long history [5,6] possible new analytical insight into the solutions of DGLAP evolution equations, if obtained, are expected to clarify the question of how much the results of such formalism can be made flexible

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enough to bring them to the close proximity of the exact results with least number of parameters. This is the aim of the present paper.

In Section 2, we record the essential formalism while Section 3 contains the results and discussions.

2. Formalism

2.1 The DGLAP equation for non-singlet structure function in next-to-leading order is [5,6]

$$\frac{\partial F^{NS}(x,t)}{\partial t}=\frac{\alpha_s(t)}{2\pi}\left[\frac{2}{3}\{3+4\log(1-x)\}F^{NS}(x,t)-\frac{4}{3}\int_x^1\frac{dz}{1-z}\left\{\left(1+z^2\right)F^{NS}(x/z,t)-2F^{NS}(x,t)\right\}\right]$$

$$+\left(\frac{\alpha_s(t)}{2\pi}\right)^2\left[(x-1)F^{NS}(x,t)\int_0^1 f(z)dz+\int_x^1 f(z)F^{NS}\left(x/z,t\right)dz\right] \tag{1}$$

with

$$f(z)=\frac{16}{9}P_F(z)+2P_G(z)+\frac{2}{3}n_f P_{N_f}(z)+\frac{2}{9}P_A(z) \tag{2}$$

where $P_F(z)$, $P_G(z)$, $P_{N_f}(z)$ and $P_A(z)$ are taken from [5,6].

In eq.(1), $t=\log\left(\frac{Q^2}{\Lambda^2}\right)$

Running coupling constant in next-to-leading order (NLO) is [5,6]

$$\alpha_s(t)=\frac{4\pi}{\beta_0 t}\left[1-\frac{\beta_1\log t}{\beta_0^2\, t}\right]$$

for one loop with

$$\beta_0=\frac{33-2n_f}{3}\quad\text{and}\quad\beta_1=\frac{306-38n_f}{3}$$

n_f being the number of flavours.

Using Taylor expansion [7–10] and using eqs. (2)-(4) of Ref. [1] we obtain from eq. (1),

$$\frac{\partial F^{NS}(x,t)}{\partial t}-\left[T(t)A(x)+T^2(t)B(x)\right]\frac{\partial F^{NS}(x,t)}{\partial x}-\left[T(t)C(x)+T^2(t)D(x)\right]F^{NS}(x,t)=0 \tag{3}$$

where

$$T(t)=\frac{\alpha_s(t)}{2\pi}=\frac{2}{\beta_0 t}\left(1-\frac{\beta_1 \log t}{\beta_0^2 t}\right) \tag{4}$$

and

$$A(x)=\frac{2}{3}\left(-2x\log x+x(1-x^2)\right) \tag{5}$$

$$B(x)=x\int_x^1 \frac{1-z}{z} f(z)\,dz \tag{6}$$

$$C(x)=\frac{2}{3}[3+4\log(1-x)+(x-1)(x+3)] \tag{7}$$

$$D(x)=-\int^{x} f(z)\,dz+x\int^{1} f(x)\,dz \tag{8}$$

where $f(z)$ is defined in eq. (2).

In order to solve eq. (3), it is recast in the standard form [3]

$$Q(x,t)\frac{\partial F^{NS}(x,t)}{\partial t}+P(x,t)\frac{\partial F^{NS}(x,t)}{\partial x}=R'(x,t) \tag{9}$$

to be compared with eq. (5) or Ref. (1).

Here,

$$Q(x,t)=1 \tag{10}$$

$$P(x,t)=-T(t)\left[A(x)+T(t)B(x)\right] \tag{11}$$

$$R(x,t)=T(t)\left[C(x)+T(t)\,D(x)\right] \tag{12}$$

$$R'(x,t)=R(x,t)\,F^{NS}(x,t). \tag{13}$$

The general solution of eq. (9), frequently referred to as Lagrange's equation [3] is obtained from the solutions of the equation [eq. (9) of Ref. [1]] *viz.*

$$\frac{dx}{P(x,t)}=\frac{dt}{Q(x,t)}=-\frac{dF^{NS}(x,t)}{R(x,t)F^{NS}(x,t)}. \tag{14}$$

Here the expressions for $P(x,t)$ and $R(x,t)$ defined in eq. (11) and eq. (12) are not factorizable in x and t unlike the corresponding equations of Ref. [1]. [eq. (6) and eq.(8)]. For a possible analytical solution, $T^2(t)$ is linearised [11] through the ansatz

$$T^2(t) \cong T_0 T(t) \tag{15}$$

where T_0 is a numerical parameter to be determined from experiment.

This results in

$$P(x,t) = -T(t)\left[A(x) + B(x)\,T_0\right] \tag{16}$$

$$R(x,t) = T(t)\left[C(x) + D(x)\,T_0\right] \tag{17}$$

The general solution of eq. (3) is

$$F\left(U_{NLO}, V_{NLO}\right) = 0 \tag{18}$$

where F is an arbitrary function and $U_{NLO}\left(x,t,F^{NS}\right) = C_1$, $V_{NLO}\left(x,t,F^{NS}\right) = C_2$ are two independent solutions of eq. (14)

(Here C_1 and C_2 are two arbitrary constants.)

The first equation of the system of eq. (14) is

$$\frac{dx}{-T(t)\left[A(x) + B(x)T_0\right]} = \frac{dt}{1} \tag{19}$$

the solutions of which is

$$U_{NLO}\left(x,t,F^{NS}\right) = t^{\left(\frac{b}{t}+1\right)} \exp\left[\frac{b}{t} + \frac{N(x)}{a}\right]. \tag{20}$$

Another equation of the same system is

$$\frac{dx}{-T(t)\left[A(x) + B(x)\,T_0\right]} = \frac{dF^{NS}(x,t)}{T(t)\left[C(x) + D(x)T_0\right]F^{NS}(x,t)} \tag{21}$$

the solution of which is

$$V_{NLO}\left(x,t,F^{NS}\right) = F^{NS}(x,t)\exp\left[M(x)\right] \tag{22}$$

where

$$a = \frac{2}{\beta_0}, \quad b = \frac{\beta_1}{\beta_0^2} \text{ and}$$

$$N(x)=\int\frac{dx}{A(x)+B(x)\,T_r} \tag{23}$$

and
$$M(x)=\int\frac{C(x)+D(x)T_0}{A(x)+B(x)T_0}dx\,. \tag{24}$$

The most general form of eq. (18) linear in F^{NS} (*i.e.* in V_{NLO}) has structure similar to eq. (19) of Ref. [1], *viz.*

$$V_{NLO}=\alpha U_{NLO}^{n'}+\beta \tag{25}$$

where α and β are two arbitrary constants $n'=n'(x,t)$ and is any real function of x and t.

From eqs. (20), (22) and (25), one gets,

$$F^{NS}(x,t)\exp[M(x)]=\alpha\left[t^{\left(\frac{b}{t}+1\right)}\exp\left\{\frac{b}{t}+\frac{N(x)}{a}\right\}\right]^{n'}+\beta\,. \tag{26}$$

Using eq. (21) of Ref. (1), *viz*

$$F^{NS}(1,t)=0 \tag{27}$$

for any t, we get

$$\beta=-\alpha\left[t^{\left(\frac{b}{t}+1\right)}\exp\left\{\frac{b}{t}+\frac{N(1)}{a}\right\}\right]^{n'}. \tag{28}$$

Putting the values of β in eq. (26), one gets

$$F^{NS}(x,t)\,\exp[M(x)]=\alpha\, t^{n'\left(\frac{b}{t}+1\right)}\exp\left[\frac{n'b}{t}\right]\left[\exp n'\left\{\frac{N(x)}{a}\right\}-\exp n'\left\{\frac{N(1)}{a}\right\}\right]. \tag{29}$$

Now defining,

$$F^{NS}(x,t_0)\,\exp[M(x)]=\alpha\, t_0^{n'\left(\frac{b}{t_0}+1\right)}\exp\left[\frac{n'b}{t_0}\right]\left[\exp n'\left\{\frac{N(x)}{a}\right\}-\exp n'\left\{\frac{N(1)}{a}\right\}\right] \tag{30}$$

we finally get,

$$F^{NS}(x,t) = F^{NS}(x,t_0)\left\{\frac{t^{n'\left(\frac{b}{t}+1\right)}}{t_0^{n'\left(\frac{b}{t_0}+1\right)}}\right\}\exp\left[n'b\left(\frac{1}{t}-\frac{1}{t_0}\right)\right]. \tag{31}$$

In Ref. [1], it has been suggested that the general solution of non-singlet structure function for low $x(x \ll 1)$ in LO of α_s is given by (eq. (26) of Ref. (1)).

$$F^{NS}(x,t) = F^{NS}(x,t_0)\left(\frac{t}{t_0}\right)^n \tag{32}$$

where $n = n\,(x,t)$ is any real function of x and t such that $n > 0$ numerically. The solution given by eq. (31) in the leading order limit $\left(b = \beta_1/\beta_0^2 \to 0\right)$ should yield eq. (32) and that is possible if $n'(x,t) = n(x,t)$.

2.2. High x solution :

Using the approximation for high x pursued in Ref. (1), we get an equation similar to eq. (3) except the function $A(x)$ defined in eq. (5) gets modified to

$$A(x) \to A'(x) = \frac{4}{9}x(1-x)\left(x^2+x+4\right). \tag{33}$$

Similarly, the functions $N(x)$ and $M(x)$ defined in eqs. (23) and (24) are also modified

$$N(x) \to N'(x) = \int \frac{dx}{A'(x) + B(x)\,T_0} \tag{34}$$

and

$$M(x) \to M'(x) = \int \frac{C(x) + D(x)T_0}{A'(x) + B(x)\,T_0}\,dx. \tag{35}$$

As in low x, demanding linearity of the solutions in $F^{NS}(x, t)$ as defined in eq. (18), we get the following form similar to eq. (25)

$$V'_{NLO} = \alpha_1 U'^{\kappa}_{NLO} + \beta_1 \tag{36}$$

where

$$U'_{NLO}(x,t,F^{NS}) = t^{\left(\frac{b}{t}+1\right)}\exp\left[\frac{b}{t} + \frac{N'(x)}{a}\right] \tag{37}$$

and

$$V'_{NLO}(x,t,F^{NS}) = F^{NS}(x,t)\exp\left[M'(x)\right]. \tag{38}$$

In eq. (36), $k' = k'(x,t)$ is any real function of x and t similar to $n' = n'(x,t)$ of eq. (31).

From eqs. (36), (37) and (38), one gets

$$F^{NS}(x,t)\exp\left[M'(x)\right] = \alpha_1\left[t^{\left(\frac{b}{t}+1\right)}\exp\left\{\frac{b}{t}+\frac{N(x)}{a}\right\}\right]^{k'} + \beta_1 \tag{39}$$

which has a term similar to eq. (26) for low x limit.

We now use the boundary conditions eq. (27) to finally obtain the t-evolution of $F^{NS}(x,t)$ at high x :

$$F^{NS}(x,t) = F^{NS}(x,t_0)\left\{\frac{t^{k'\left(\frac{b}{t}+1\right)}}{t_0^{k'\left(\frac{b}{t_0}+1\right)}}\right\}\exp\left[k'b\left(\frac{1}{t}-\frac{1}{t_0}\right)\right]. \tag{40}$$

As the general solution of non-singlet structure function for high x ($x \to 1$) in LO is (eq. (47) of Ref. (1)]

$$F^{NS}(x,t) = F^{NS}(x,t_0)\left(\frac{t}{t_0}\right)^{p(x,t)}, \qquad p(x,t) < 0\,. \tag{41}$$

The solution eq. (40) in the leading order limit ($b \to 0$) must conform to eq. (41), one again has the equality

$$k'(x,t) = p(x,t)\,.$$

We can also reexpress eq. (31) and eq. (40) in the following form

$$F^{NS}(x,t) = F^{NS}(x,t_0)\left(\frac{t}{t_0}\right)^{H_{NLO}(x,t)} \tag{42}$$

where

$$H_{NLO}(x,t) = \frac{n(x,t)\log\left\{\dfrac{t^{\left(\frac{b}{t}+1\right)}}{t_0^{\left(\frac{b}{t_0}+1\right)}}\right\} + n(x,t)\,b\left(\dfrac{1}{t}-\dfrac{1}{t_0}\right)}{\log\left(\dfrac{t}{t_0}\right)} \tag{43}$$

for $x \to 0$ and

$$H_{NLO}(x,t) = \frac{p(x,t)\log\left\{\dfrac{t^{\left(\frac{b}{t}+1\right)}}{t_0^{\left(\frac{b}{t_0}+1\right)}}\right\} + p(x,t)\,b\left(\dfrac{1}{t}-\dfrac{1}{t_0}\right)}{\log\left(\dfrac{t}{t_0}\right)} \tag{44}$$

for high $x(x \to 1)$ limits respectively.

The ratio of H_{NLO} to n (for low x) and p (for high x) is defined as $R(t)$:

$$R(x,t) = \frac{\log\left\{\dfrac{t^{\left(\frac{b}{t}+1\right)}}{t_0^{\left(\frac{b}{t_0}+1\right)}}\right\} + b\left(\dfrac{1}{t}-\dfrac{1}{t_0}\right)}{\log\left(\dfrac{t}{t_0}\right)} \tag{45}$$

yields

$$H_{NLO}(x,t) = H_{LO}(x,t)\,R(t) \tag{46}$$

where for $x \to 0$, $H_{LO}(x,t) \to n(x,t)$ and for $x \to 1$, $H_{LO}(x,t) \to p(x,t)$.

As $R(t)$ is positive for $t > t_0$ as shown in Figure (1), it implies that the exponent $H_{NLO}(x,t)$ defined in eq. (42) has features identical to those of $n(x,t)$ and $p(x,t)$ defined earlier (in eqs. (32) and (41) respectively) *i.e.* $n(x,t) > 0$ for low x and $p(x,t) < 0$ for high x. This suggests that even in the next-to-leading order, the present formalism enables one qualitatively to separate out low x and high x regimes explored in deep inelastic scattering It is worthwhile to note that the main results of the present work eq (31) and eq (40) are improvement over those of Ref. (11) as the present form of evolutions are both T_0 and x independent, which were not so there. It is also more general than those of Ref (12) based on adhoc relation $\beta = \alpha^{\gamma}(\gamma = 2,3,4,5 \ldots\ldots)$

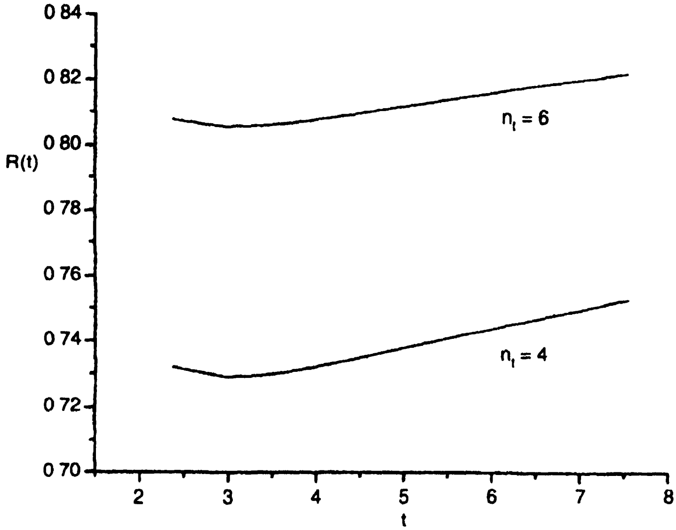

Figure 1. We plot $R(t)$ (eq 45) *vs* t for two distinct quark flavours $n_f = 4$ and $n_f = 6$ respectively

3. Results and discussions

In order to make phenomenological analysis, we use eq (42) to define $H_{\text{expt}}(x,t)$ as

$$H_{\text{expt}}(x,t) = \frac{\log F^{\text{expt}}(x,t) - \log F(x,t_0)}{\log\left(\frac{t}{t_0}\right)}. \tag{47}$$

The numerical value of $H_{\text{expt}}(x,t)$ will be obtained by the analysis of recent CCFR experiment [4] and taking $\Lambda = 323$ MeV.

Similarly we can define $H_{\text{exact}}(x, t)$ as

$$H_{\text{exact}}(x,t) = \frac{\log F^{\text{exact}}(x,t) - \log F(x,t_0)}{\log\left(\frac{t}{t_0}\right)} \tag{48}$$

which is obtained from HEP data base [13] using MRST 2004 NLO parton distributions [13].

In Figure 2(a-d), we plot $H_{expt}(x, t)$ *vs* Q^2 for four representative values of low x : x = 0.0075, 0.0125, 0.0175, 0.0250. It shows that most of the data for it is found to be positive as suggested by the theory. However, in the following few (x, Q^2) values, $H_{expt}(x,t)$ for low x turns out to be negative : (0.0075, 1 3), (0.0075, 3.2), (0.00125, 3.2), (0.0175, 1.3), (0.0250, 1 3), which is anomalous in the context of the present formalism.

For high x, we make similar analysis for four representative values of x : x = 0.4500, 0.5500, 0.6500, 0.7500 as in Figure 2(e-h). In this case, however $H_{expt}(x,t)$ comes out to be invariably negative, suggesting the validity of the present formalism.

In Ref [1], we have suggested most general form of $H_{LO}(x,t)$ while a simple empirical form of it was found to be

$$H^{LO}_{expt}(x,t) = 0\,380[-5\,796x + 0\,996(1-x)] \tag{49}$$

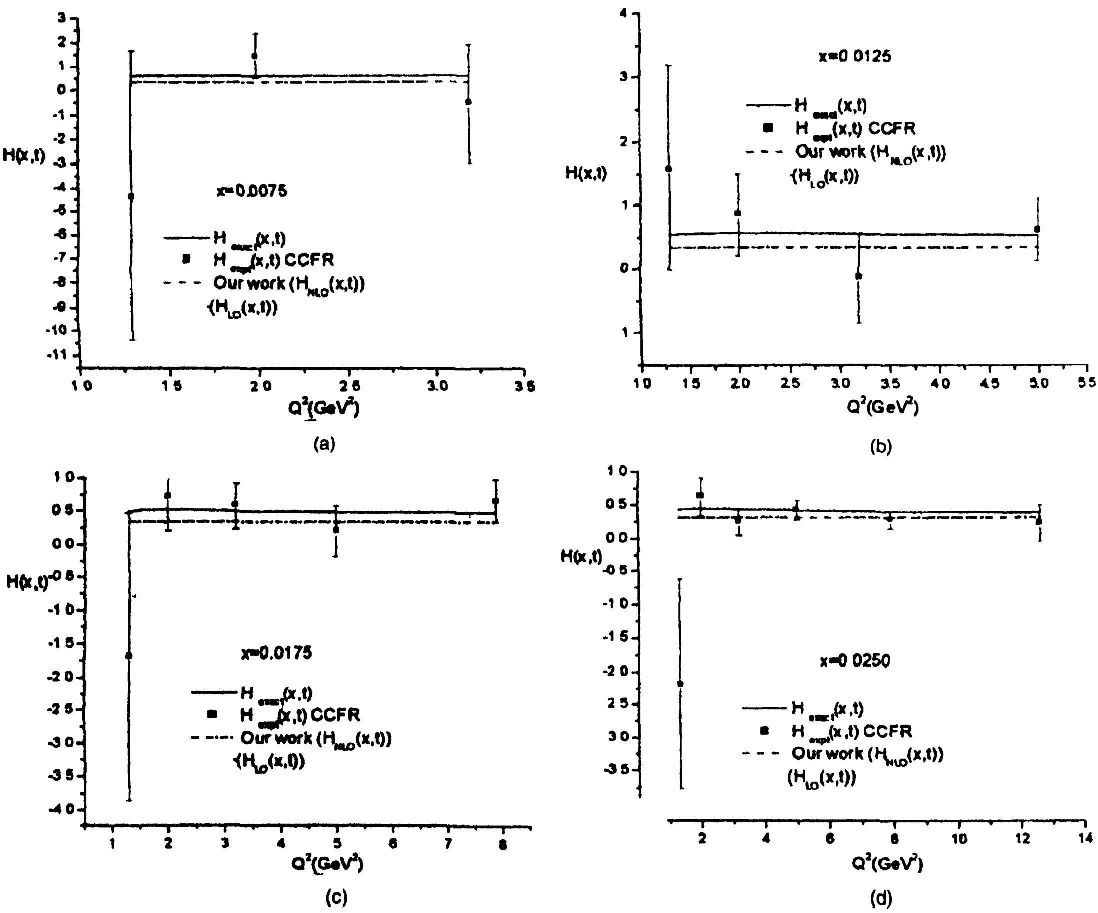

Figure 2 (a-d). We plot $H_{expt}(x,t)$ *vs* Q^2 for four representative values of low x : x = 0.0075, 0.0125, 0 0175, 0.0250 For high x, we make similar plotting for four representative values of x : x = 0 4500, 0.5500, 0.6500, 0.7500 in Figure 2(e-h).

A reanalysis of the data with NLO MRST distributions [13] modifies eq. (49) to

$$H_{\text{expt}}^{\text{NLO}} = 0.412\,[-4.51x + 0.913(1-x)] \tag{50}$$

In Figure 2(a–d), we compare for low x the predictions of eq. (49) (dotted line) and eq. (50) (dashed dot line) with exact results eq. (48) (solid line) while in Figure 2(e–f) a similar comparison is done for high x. For low x, the NLO effects of eq. (50) are found to be negligible as its predictions nearly coincide with those of eq. (49). However, for high x, the predictions of eq. (50) are closer to exact one eq. (48) except for the point x = 0.4500 where eq. (49) is favoured.

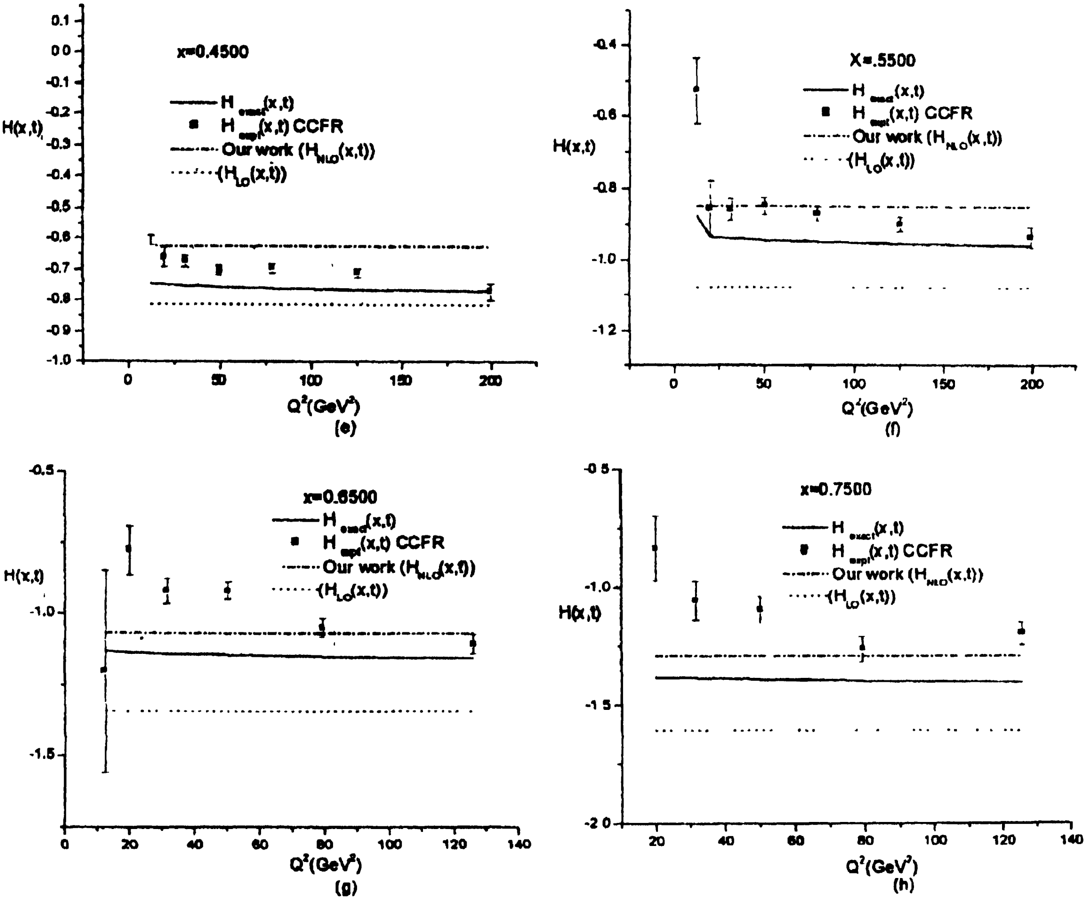

Figure 2 (e–h). We compare for low x the predictions of eq. (49) (dotted line) and eq.(50) (dashed dot line) with exact results eq. (48) (solid line) while in Figure 2(e–h) a similar comparison is done for high *x*.

4. Comments and conclusions

To conclude we have shown that the exponent of (t/t_0) in the expression for the structure function in the next-to-leading order is positive for low x and is negative for high x, as in leading order. Through our formalism, we have been able to separate the low and high x

region respectively. In contrast to the result of the previous work (Refs. 7–11), we have suggested that the exact form of the exponent can be much more general than the mere geometrical numbers ±1. As its exact form is not given by the theory, we suggested a general structure and obtained a simple empirical form consistent with CCFR data. Our analysis above suggests that such empirical form with NLO effects is closer to exact results for most of the data at high x, while for low x, such effects are insignificant.

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